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Collective cavity mode excitations in arrays of Josephson junctions

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We report on oscillations of a Josephson-junction system revealing a close analogy with fundamental effects known in laser physics. The experiments are performed on series arrays of junctions whose *IV* curves show evidence of a mode in which all the junctions oscillate in synchronism on resonances appearing, in zero external magnetic field, at multiples of the first cavity mode. Evidence is provided that the mode is generated by collective oscillations which develop spontaneously because the frequencies of the modes are equal within a 1% uncertainty. Numerical simulations are employed to elucidate the reasons leading to the synchronous collective excitation and to explain why it is characterized by a low emission of electromagnetic radiation.

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The macroscopic evidence of phenomena ruled by quantum mechanics at microscopic level is a subject which has puzzled scientists working in several fields of physics among which optics,[1] magnetism,[2] and superconductivity.[3] The interest for this fundamental issue nowadays is much strengthened by the development of quantum computation research,[4] a field in which the macroscopic properties of the superconducting wave functions play a relevant role. Most of the reported experiments on Josephson phase and flux qubit systems have indeed evident links with nuclear magnetism[5] and optics;[6] also, the synchronization of arrays of Josephson junctions is a rather interesting issue in the physics of complex systems[7,8] but also in applied science for potential millimeter and submillimeter wave devices.[9]

The cavity modes-based dynamics of long Josephson junctions was systematically characterized in connection with the explanation of Fiske steps;[10] a relevant feature of this dynamics is the fact that a threshold dc magnetic field is required for the activation of stable Fiske steps. The Fiske modes are generated by the interaction of the ac Josephson effect at multiples of the fundamental cavity mode frequency $\nu_{FS}=(\frac{\bar{c}}{2L})$, where $\bar{c}$ is the speed of light in the oxide barrier and L is the length of the junctions along which the oscillations take place.[10] It has also been shown that a junction biased on the cavity modes generating Fiske modes can phase lock to a boundary external time-varying field for substantial current intervals.[11] In this paper we will show that a series array of long junctions can develop, even in the absence of an external magnetic field, a collective excitation in which all the junctions oscillate at the frequency of the first cavity mode absorbing most of the energy available in the system.

We work here on arrays of hundreds of Josephson junctions connected in series in a meander shape: two neighbor lines are shown in Fig. 1(a). The fabrication process, relying on the basic Nb/Al-AlO_x/Nb junction trilayer technology is described elsewhere.[12] However, for the present experiments we used a mix and match technology for optical and electron-beam lithography; in particular, the areas of the junctions were defined by electron-beam exposure. In Figs. 1(b) and 1(c) we show current-voltage (*IV*) characteristics of two arrays of long inline junctions.[13] We see that the *IV*s present a rather anomalous effect, namely, a linear "slope" in the switching currents distribution. The long junctions had a symmetric inline geometry[13,14] with only one physical dimension larger than the Josephson penetration depth λ_j (junction dimensions are "long" or "short" for comparison with this parameter).[13] The current-voltage characteristics of the samples shown in Figs. 1(b) and 1(c) are relative to arrays with a different number of junctions and produced in different fabrication batches; in particular, we had for (b) J_c=190 A/cm^2 and for (c) J_c=470 A/cm^2. The long junctions in (b) and (c) had a length of 54.5 μm corresponding, respectively, to 1.5λ_j, and 2.3λ_j and both had a width of 5 μm. The measurements shown in Figs. 1(b) and 1(c) were performed at 4.2 K in a liquid-helium bath in a double magnetically shielded environment: we had a "cold" cryoperm shield at 4.2 K around the samples and a room-temperature mumetal shield surrounding the liquid-helium dewar.

We observed the same features shown in the Figs. 1(b) and 1(c) for junctions lengths up to 75 μm and for several different current densities. These peculiarities, however, are not observed on long junctions arrays with identical geometry fabricated by fully optical lithography [see inset of Fig. 1(b)] or on small Josephson junctions arrays fabricated by mix and match technology [see inset of Fig. 1(c)]. Our conclusion is that the phenomenon reported in Fig. 1 is not generated by a specific current density. Also, the specific geometrical configuration of our chips generating a peculiar electromagnetic coupling between the junctions is not, by itself, responsible for the effect since identical chips fabricated with fully optical lithography were very regular. The effect relies on the junctions being larger than λ_j (which can cause a strong interaction of the cavity modes with the Josephson effect) and on the fact that the dimensions of all the

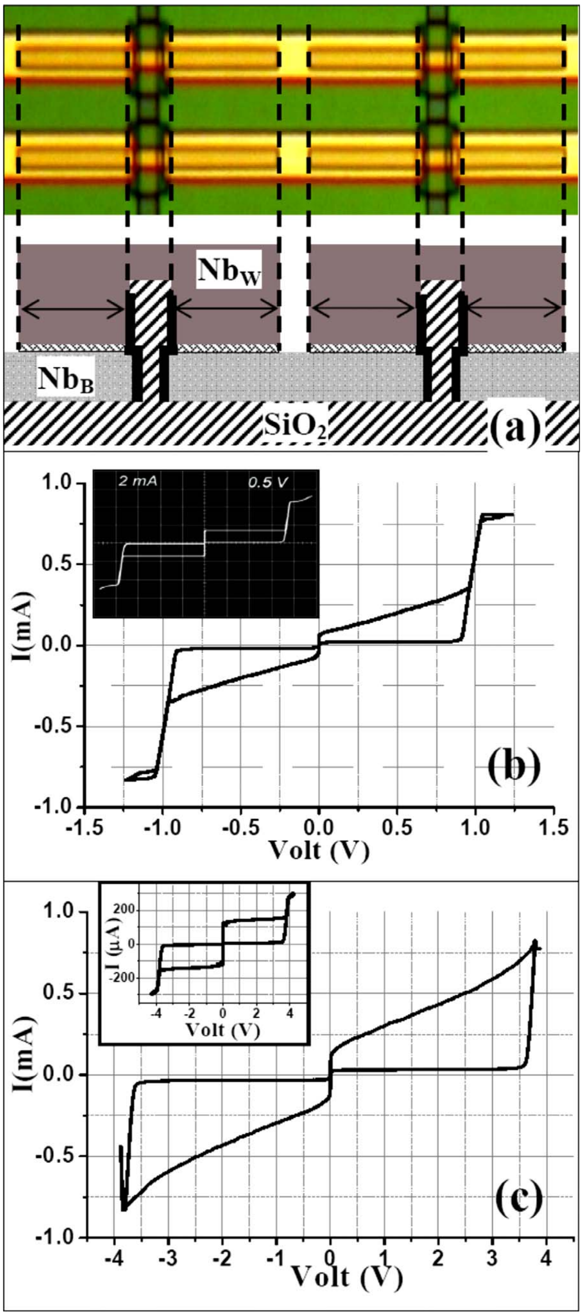

FIG. 1. (Color online) (a) A zoom of our series arrays (photo, above) and a cross section (below): Nb_B and Nb_W stands for niobium base and wiring electrodes of the junctions whose length is indicated by the double arrowed lines; (b) current-voltage characteristics of long Josephson junctions arrays containing 374 junctions displaying an "anomalous" increase in the switching currents. The inset shows the "ideal" *IV* curve of long junctions array fabricated by a fully optical lithography. (c) An array formed by 1394 junctions; the inset here shows the characteristics of a small junctions array placed on the same chip.

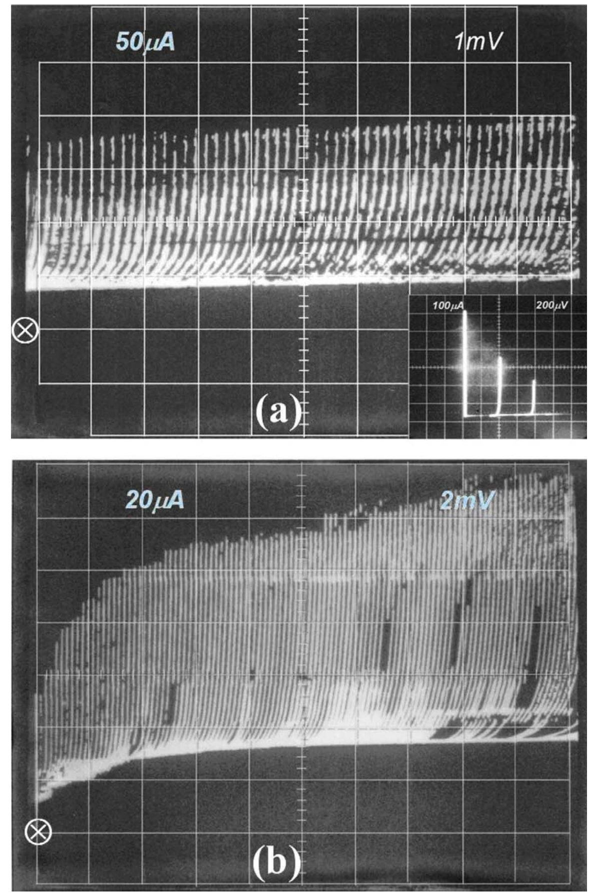

FIG. 2. (Color online) Sequences of zero-field Fiske steps obtained for a series array, respectively, of (a) 374 and (b) 1394 junctions. The inset shows the result of biasing only one 54.5-μm-long junction of an array: here we only see the zero-field steps spaced twice the voltage of the Fiske steps. Markers ($\otimes$) indicate axes origins.

junctions are defined with electron-beam exposure which allows to achieve a very low spread over the nominal linear dimensions of the junctions. Over the junctions lengths we estimate, for example, a spread on the order of (0.5–1)% which is roughly a factor 5 better than what can be done by the standard UV optical lithography.

Superimposing a dc voltage offset to the low-frequency current sweep feeding the junctions for recording the *IV* curves like those shown in Fig. 1, we obtained the zoom of the current-voltage characteristics shown in Fig. 2, which reveals the real origin of the "switching currents" shown in Figs. 1(b) and 1(c). The array of Fig. 2(a) had junctions with the same length of those of Fig. 1(b) but a current density 2.5 times greater. The sample of Fig. 1(c) instead is the same of that shown in Fig. 2(b); note when comparing the current scales of these two figures that three orders of magnitude difference exists in the voltage axes. The presence of equally spaced resonances and the switchings from the maximum current of these produce a stairway which, displayed in a compressed scale, generates the linear slope shown in Figs. 1(b) and 1(c); the voltage spacing of the singularities corresponds to that of the Fiske step (first cavity mode resonance with Josephson effect) of individual junctions.[10] Indeed, from the position of Fiske modes and zero-field steps[13] in individual long and short junctions we calculated that the speed of light in the oxide barrier is $\bar{c}=0.035c$ and therefore the spacing of the Fiske steps, for a junction 54.5 μm long, should be 200 μV like we see in Fig. 2. Considered that the resonances of Fig. 2 exist in zero magnetic field, these are zero-field Fiske steps; Camerlingo *et al.*[13] reported on these singularities for peculiar current distribution while Kautz[9] showed that Fiske modes can be generated by rf fields in absence of a dc magnetic field.

We found that a reasonable estimate of the slopes of the straight lines of Fig. 1 is given by $(I_{MOS}\text{-}I_{M1FS})/NVg$, where I_{MOS} is the maximum theoretical Owen-Scalapino current for symmetric inline junctions (which we can calculate with a 10% uncertainty),[14] I_{M1FS} the maximum current of the first Fiske step and NVg the sum of all the gaps of all the N junctions forming the array. In other terms the minimal current corresponds to that of a Fiske mode of a single junction while the maximum current of the singularities is the maximum attainable dc Josephson current in each junction; I_{MOS} and I_{M1FS} can be readily calculated from the parameters of the junctions.[10,14] For Fig. 1(b) [respectively, Fig. 1(c)] I_{MOS}=350 μA (respectively, I_{MOS}=750 μA) and the maximum height of the first Fiske mode of isolated junctions of the same length was I_{M1FS}=70 μA (respectively, I_{M1FS} =160 μA).

The voltages of the singularities shown in Fig. 2 add up to the sum-gap voltage of the series array, meaning that the mode of oscillation at 97 GHz (the Josephson frequency corresponding to 200 μV) is shared by all the junctions in the array. Our above "ansatz" for the slope naturally does not explain why the linear relation between voltage and current height of the resonances exists. We note, however, that the fact that in the *IV* curves the maximum current amplitude of the singularities increases linearly with the number of junctions N implies, since the voltages of the series array also depend linearly on N, that the power associated with the height of the singularities increases like N^2. This specific feature has been observed in other phase-locked Josephson systems[8,9] but the locking on cavity modes in our array is fully autonomous and not forced through stripline resonators, feedback loads, or external microwave fields. In spite of the possible internal coherence of the Josephson cavities modes that can be argued from the *IV* curves, very little power was coupled out of the arrays while substantial amounts of radiation were coupled from arrays fabricated with "fully optical" technique. The power was measured through a detector array[15] coupled to the long junctions array. On the "fully optically" fabricated long junctions arrays, on an expanded voltage scale such that shown in Fig. 2, usual zero-field steps or (under application of an external field) Fiske steps[15] were observed and radiation was detected when dc biased on these.

From the experimental evidences we conclude that decreasing the scatter of the frequencies of the Josephson cavity oscillators (by the mix and match technique) the whole array develops internal oscillations which absorb much of the input bias energy and leave little to be detected out. This phenomenon has a striking analogy in laser physics:[16] in a laser cavity, decreasing the transmittance of the mirrors, the output power can be very low because most energy remains in the cavity (the output power has a maximum indeed for an "optimal" value of the transmittance).

In order to perturb the spontaneous internal oscillations giving rise to the *IV* curves of Figs. 1(b) and 1(c) we applied an external magnetic field (perpendicular to the long side of the junctions); such a field breaks the phase symmetry generated by the dc-bias current at the ends of each junction.[10,17] The result is shown in Fig. 3 for a chip with current density equal to that of Fig. 1(c). We see here in (a) that the slopes decrease rapidly by increasing the field and in (b) we report the result of the dependence of the measured slopes of the linear portions of the *IVs* in (a) as a function of the applied magnetic field (continuous curve). In spite of the lowered values of the dc currents a substantial amount of power, like in Ref. 15, is detected out when biased on the *IV* current singularities where the slope has disappeared; in laser cavity language we can say that the external field has slightly detuned the internal synchronization allowing more input power to leak out as radiation.

A relevant physical insight of the dependence shown in (b) can be gained looking at the derivative of the dashed curve (Gaussian fit to the experimental data): we can see that the value of the field for which the derivative has the maxi-

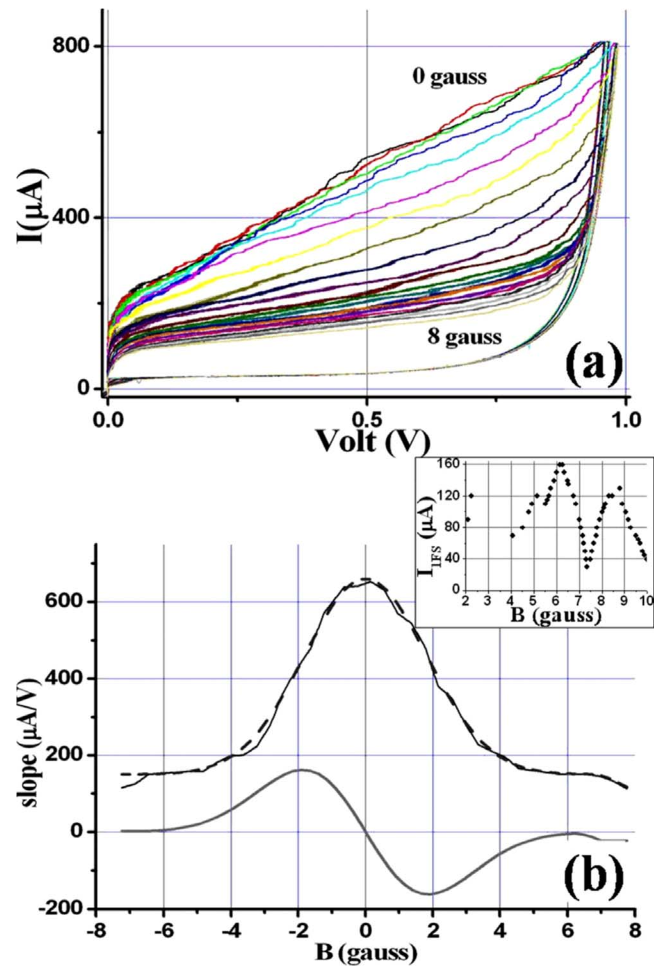

FIG. 3. (Color online) (a) The dependence of the *IV* curves upon the external magnetic field; (b) the slope of the linear portions of the *IVs* in (a) vs the external magnetic field. Continuous line are the experimental data, the dashed line is a Gaussian fit to the data performed in order to obtain a "smooth" derivative [reported below in (b)]. The inset shows the modulations of stable Fiske steps of the junction of the inset of Fig. 2(a).

mum is 1.9 G. We recall now that the stable Fiske modulations take place in long junctions when, above the value $H_0=2\lambda_j J_c$ the magnetic field penetrates in the junctions;[10] below this value, due to boundary effects, flux-quanta dynamics can take place.[10,17] The calculation of H_0 for the sample of Fig. 2(a) from the values of J_c and λ_j gives $(1.9\pm 5\%)$ G. Our conclusion is that the cavity modes of Fig. 2 are triggered by slight phase gradients generated by the bias current at the ends of the inline junctions and therefore the effect disappears above H_0.

We were successful in contacting (by a careful on-chip bonding) one single junction of the array of Fig. 1(c) which could then be dc biased without feeding all the others. The *IV* curve of this junction did not show zero-field Fiske steps: a particular of the *IV* characteristics of this isolated junction is the one shown in the inset of Fig. 2(a) where we see the Josephson current and two zero-field steps (soliton modes)[13] spaced 400 μV (twice the voltage of the Fiske steps, as expected). Also, we see in the inset of Fig. 3(b) that in this same junction Fiske steps can be recorded right after 2 G as expected from the above value of H_0: this is a further confirmation that the maximum of the derivative shown in Fig. 3(b) corresponds indeed to a change in dynamical regime.

In order to investigate the dynamical model we performed numerical simulations coupling the junctions capacitively at the ends,[18] a reasonable approximation for modeling the rf coupling of junctions belonging to the same superconducting island [base or wiring, see Fig. 1(a)]. We imposed as initial conditions one flux quantum in each junction moving with an initial power balance velocity 0.92 (Ref. 13) corresponding to a bias current (normalized to the maximum Owen-Scalapino current[14]) of 0.3 and a McCumber parameter[13] $\beta_c=100$ (a value chosen for accelerating the numerical convergence). The experiments showed that flux-quanta oscillations of individual junctions do not survive the reflections and therefore a value of 0.5 was chosen for the normalized coupling capacitance.[18] In Fig. 4(a) we show the spatial derivative of the phase (surface current) taken along four coupled junctions each long 3 (space normalized to λ_j): the four junctions, after an initial disordered transient, synchronize on internal cavity modes. In Fig. 4(a) the capacitors, whose position is indicated by the arrows on the axes, coupling the four junctions are all three equal and we found the oscillations stable within a few percent difference between the lengths of the junctions.

In Fig. 4(b) we show the case in which the value of the central capacitor is set to 0.1 (leaving the others to 0.5). The difference between the capacitors is a reasonable hypothesis for the circuit configuration [see Fig. 1(a)] since the two ends of each junction face a different electromagnetic environment: at one end we have SiO_2 ($\varepsilon_r=4.2$) while at the other we have essentially liquid helium ($\varepsilon_r=1.05$). We see in Fig. 4(b) that for this set of parameters the junctions result coupled "in pairs" on cavity modes: the couple of junctions (total length 6) beside the central capacitor oscillate on the second cavity mode (note the zeros at the crossing of the two waveforms at $x=2,4,8,10$) while each single junction oscillates on the first cavity mode. The current oscillations have a larger amplitude than those shown in Fig. 4(a) and the instantaneous power is much increased [see Fig. 4(c)] but in each junction the oscillation pattern corresponding to the first Fiske step can develop. The oscillations of Fig. 4(b) were stable only within 1% difference between the lengths of the junctions and were negatively influenced by an external dc magnetic field, similarly to what we see in experiments; we found that even a field, normalized to $H_0/2$,[10] equal to 0.1 could dephase the oscillators.

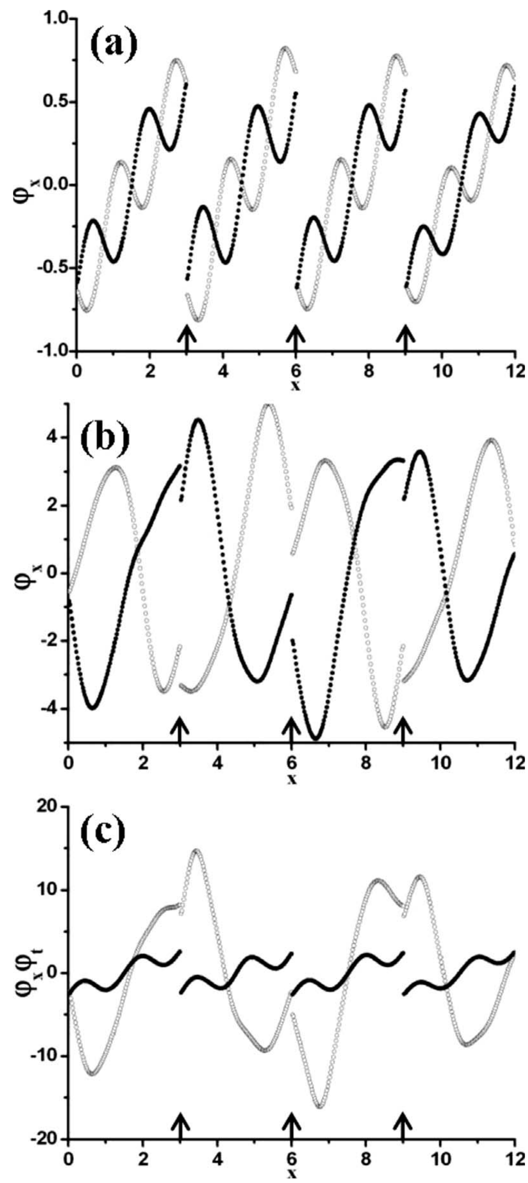

FIG. 4. (a) Two-time shots for the synchronized third-order cavity modes of four coupled inline junctions; (b) two-time shots for junctions coupled in pairs on the second-order cavity mode, the first mode of each individual junction; (c) two instantaneous pictures of the instantaneous power in the two above phase-locked modes: full circles is the mode shown in (a) while empty circles represent the mode shown in (b). The locking in pairs (empty circles) gives rise to a much increased spatial power distribution.

The fact that a large amount of power is absorbed to pump the internal modes explains why in the experiments very little power is coupled out of our arrays when biased on the modes of Fig. 2; the simulations also show that the "internal" power increases but all the junctions oscillate on the first cavity mode meaning that the voltage sums of the resonances of the array will just be a linear function of the number of biased junctions.

In conclusion we have shown how close is Josephson effect physics with phenomena having relevant and broad interest from the fundamental and applied physics point of view; the striking analogies show that the Josephson effect provides a reliable background for investigation of macroscopic phenomena related to coherence and synchronization of complex systems having in common a basic quantum-mechanical nature.

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